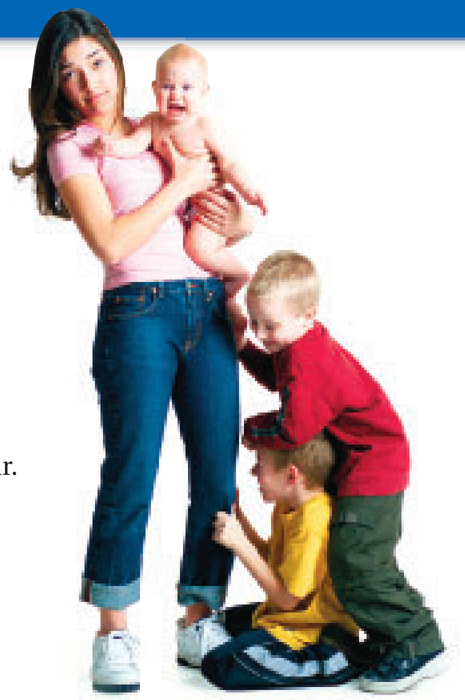
4-3 Writing Functions



Objectives
Identify independent and dependent variables.

Write an equation in function notation and evaluate a function for given input values.

Vocabulary
independent variable
dependent variable
function rule
function notation

Why learn this?
You can use a function rule to calculate how much money you will earn for working specific amounts of time.

Suppose Tasha baby-sits and charges $5 per hour.

Time Worked (h) x	1	2	3	4
Amount Earned ($) y	5	10	15	20

The amount of money Tasha earns is $5 times the number of hours she works. Write an equation using two different variables to show this relationship.

Amount earned is $5 times the number of hours worked.

$$y = 5 \cdot x$$

Tasha can use this equation to find how much money she will earn for any number of hours she works.

EXAMPLE 1 Using a Table to Write an Equation

Determine a relationship between the x- and y-values. Write an equation.

x	1	2	3	4
y	−2	−1	0	1

Step 1 List possible relationships between the first x- and y-values.

$1 - 3 = -2$ or $1(-2) = -2$

Step 2 Determine if one relationship works for the remaining values.

$2 - 3 = -1$ ✓ $2(-2) \neq -1$ ✗

$3 - 3 = 0$ ✓ $3(-2) \neq 0$ ✗

$4 - 3 = 1$ ✓ $4(-2) \neq 1$ ✗

The first relationship works. The value of y is 3 less than x.

Step 3 Write an equation.

$y = x - 3$ *The value of y is 3 less than x.*

CHECK IT OUT!

1. Determine a relationship between the x- and y-values in the relation $\{(1, 3), (2, 6), (3, 9), (4, 12)\}$. Write an equation.

The equation in Example 1 describes a function because for each x-value (input), there is only one y-value (output).

The **input** of a function is the **independent variable**. The **output** of a function is the **dependent variable**. The value of the dependent variable *depends* on, or is a function of, the value of the independent variable. For Tasha, the amount she earns depends on, or is a function of, the amount of time she works.

EXAMPLE 2 Identifying Independent and Dependent Variables

Helpful Hint

There are several different ways to describe the variables of a function.

Independent Variable	Dependent Variable
x-values	y-values
Domain	Range
Input	Output
x	$f(x)$

Identify the independent and dependent variables in each situation.

A In the winter, more electricity is used when the temperature goes down, and less is used when the temperature rises.

The amount of electricity used *depends on* the temperature.

Dependent: amount of electricity   Independent: temperature

B The cost of shipping a package is based on its weight.

The cost of shipping a package *depends on* its weight.

Dependent: cost   Independent: weight

C The faster Ron walks, the quicker he gets home.

The time it takes Ron to get home *depends on* the speed he walks.

Dependent: time   Independent: speed

CHECK IT OUT! **Identify the independent and dependent variables in each situation.**

2a. A company charges $10 per hour to rent a jackhammer.

2b. Camryn buys p pounds of apples at $0.99 per pound.

An algebraic expression that defines a function is a **function rule**. $5 \cdot x$ in the equation about Tasha's earnings is a function rule.

If x is the independent variable and y is the dependent variable, then **function notation** for y is $f(x)$, read "f of x," where f names the function. When an equation in two variables describes a function, you can use function notation to write it.

The dependent variable	**is**	**a function of**	**the independent variable**.
y	**is**	**a function of**	x.
y	$=$	f	(x)

Since $y = f(x)$, Tasha's earnings, $y = 5x$, can be rewritten in function notation by substituting $f(x)$ for y: $f(x) = 5x$. Sometimes you will see functions written using y, and sometimes you will see functions written using $f(x)$.

EXAMPLE 3 Writing Functions

Identify the independent and dependent variables. Write a rule in function notation for each situation.

A A lawyer's fee is $200 per hour for her services.

The fee for the lawyer depends on how many hours she works.

Dependent: fee   Independent: hours

Let h represent the number of hours the lawyer works.

The function for the lawyer's fee is $f(h) = 200h$.

Identify the independent and dependent variables. Write a rule in function notation for each situation.

B **The admission fee to a local carnival is \$8. Each ride costs \$1.50.**

The total cost depends on the number of rides ridden, plus \$8.

Dependent: total cost Independent: number of rides

Let r represent the number of rides ridden.

The function for the total cost of the carnival is $f(r) = 1.50r + 8$.

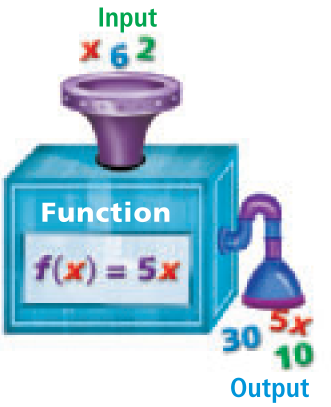
CHECK IT OUT! **Identify the independent and dependent variables. Write a rule in function notation for each situation.**

3a. Steven buys lettuce that costs \$1.69/lb.

3b. An amusement park charges a \$6.00 parking fee plus \$29.99 per person.

You can think of a function as an input-output machine. For Tasha's earnings, $f(x) = 5x$, if you input a value x, the output is $5x$.

If Tasha wanted to know how much money she would earn by working 6 hours, she could input 6 for x and find the output. This is called *evaluating the function.*



EXAMPLE 4

Evaluating Functions

Reading Math

Functions can be named with any letter; *f*, *g*, and *h* are the most common. You read $f(6)$ as "*f* of 6," and $g(2)$ as "*g* of 2."

Evaluate each function for the given input values.

A **For $f(x) = 5x$, find $f(x)$ when $x = 6$ and when $x = 7.5$.**

$f(x) = 5x$

$f(6) = 5(6)$ *Substitute 6 for x.*

$= 30$ *Simplify.*

$f(x) = 5x$

$f(7.5) = 5(7.5)$ *Substitute 7.5 for x.*

$= 37.5$ *Simplify.*

B **For $g(t) = 2.30t + 10$, find $g(t)$ when $t = 2$ and when $t = -5$.**

$g(t) = 2.30t + 10$

$g(2) = 2.30(2) + 10$

$= 4.6 + 10$

$= 14.6$

$g(t) = 2.30t + 10$

$g(-5) = 2.30(-5) + 10$

$= -11.5 + 10$

$= -1.5$

C **For $h(x) = \frac{1}{2}x - 3$, find $h(x)$ when $x = 12$ and when $x = -8$.**

$h(x) = \frac{1}{2}x - 3$

$h(12) = \frac{1}{2}(12) - 3$

$= 6 - 3$

$= 3$

$h(x) = \frac{1}{2}x - 3$

$h(-8) = \frac{1}{2}(-8) - 3$

$= -4 - 3$

$= -7$

CHECK IT OUT! **Evaluate each function for the given input values.**

4a. For $h(c) = 2c - 1$, find $h(c)$ when $c = 1$ and $c = -3$.

4b. For $g(t) = \frac{1}{4}t + 1$, find $g(t)$ when $t = -24$ and $t = 400$.

When a function describes a real-world situation, every real number is not always reasonable for the domain and range. For example, a number representing the length of an object cannot be negative, and only whole numbers can represent a number of people.

EXAMPLE 5 Finding the Reasonable Domain and Range of a Function

Manuel has already sold $20 worth of tickets to the school play. He has 4 tickets left to sell at $2.50 per ticket. Write a function rule to describe how much money Manuel can collect from selling tickets. Find a reasonable domain and range for the function.

Money collected from ticket sales	is	$2.50	per	ticket	plus	the $20 already sold.
$f(x)$	$=$	$2.50	$\cdot$	x	$+$	20

If he sells x more tickets, he will have collected $f(x) = 2.50x + 20$ dollars.

Manuel has only 4 tickets left to sell, so he could sell 0, 1, 2, 3, or 4 tickets. A reasonable domain is {0, 1, 2, 3, 4}.

Substitute these values into the function rule to find the range values.

x	0	1	2	3	4
$f(x)$	$2.50(0) + 20 = 20$	$2.50(1) + 20 = 22.50$	$2.50(2) + 20 = 25$	$2.50(3) + 20 = 27.50$	$2.50(4) + 20 = 30$

A reasonable range for this situation is {$20, $22.50, $25, $27.50, $30}.

CHECK IT OUT!

5. The settings on a space heater are the whole numbers from 0 to 3. The total number of watts used for each setting is 500 times the setting number. Write a function rule to describe the number of watts used for each setting. Find a reasonable domain and range for the function.

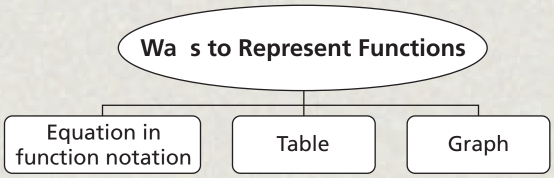
THINK AND DISCUSS

1. When you input water into an ice machine, the output is ice cubes. Name another real-world object that has an input and an output.

2. How do you identify the independent and dependent variables in a situation?

3. Explain how to find reasonable domain values for a function.

4. **GET ORGANIZED** Copy and complete the graphic organizer. Use the rule $y = x + 3$ and the domain {−2, −1, 0, 1, 2}.



4-3 Exercises

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Homework Help Online
KEYWORD: MA7 4-3
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. The output of a function is the ___?___ variable. (*independent* or *dependent*)

2. An algebraic expression that defines a function is a ___?___. (*function rule* or *function notation*)

SEE EXAMPLE 1 p. 245

Determine a relationship between the *x*- and *y*-values. Write an equation.

3.

x	1	2	3	4
y	−1	0	1	2

4. $\{(1, 4), (2, 7), (3, 10), (4, 13)\}$

SEE EXAMPLE 2 p. 246

Identify the independent and dependent variables in each situation.

5. A small-size bottle of water costs \$1.99 and a large-size bottle of water costs \$3.49.

6. An employee receives 2 vacation days for every month worked.

SEE EXAMPLE 3 p. 246

Identify the independent and dependent variables. Write a rule in function notation for each situation.

7. An air-conditioning technician charges customers \$75 per hour.

8. An ice rink charges \$3.50 for skates and \$1.25 per hour.

SEE EXAMPLE 4 p. 247

Evaluate each function for the given input values.

9. For $f(x) = 7x + 2$, find $f(x)$ when $x = 0$ and when $x = 1$.

10. For $g(x) = 4x - 9$, find $g(x)$ when $x = 3$ and when $x = 5$.

11. For $h(t) = \frac{1}{3}t - 10$, find $h(t)$ when $t = 27$ and when $t = -15$.

SEE EXAMPLE 5 p. 248

12. A construction company uses beams that are 2, 3, or 4 meters long. The measure of each beam must be converted to centimeters. Write a function rule to describe the situation. Find a reasonable domain and range for the function. (*Hint*: 1 m = 100 cm)

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–14	1
15–16	2
17–19	3
20–22	4
23	5

Extra Practice
Skills Practice p. S10
Application Practice p. S31

Determine a relationship between the *x*- and *y*-values. Write an equation.

13.

x	1	2	3	4
y	−2	−4	−6	−8

14. $\{(1, -1), (2, -2), (3, -3), (4, -4)\}$

Identify the independent and dependent variables in each situation.

15. Gardeners buy fertilizer according to the size of a lawn.

16. The cost to gift wrap an order is \$3 plus \$1 per item wrapped.

Identify the independent and dependent variables. Write a rule in function notation for each situation.

17. To rent a DVD, a customer must pay \$3.99 plus \$0.99 for every day that it is late.

18. Stephen charges \$25 for each lawn he mows.

19. A car can travel 28 miles per gallon of gas.

Evaluate each function for the given input values.

20. For $f(x) = x^2 - 5$, find $f(x)$ when $x = 0$ and when $x = 3$.

21. For $g(x) = x^2 + 6$, find $g(x)$ when $x = 1$ and when $x = 2$.

22. For $f(x) = \frac{2}{3}x + 3$, find $f(x)$ when $x = 9$ and when $x = -3$.

LINK

Transportation

Air Force One refers to two specially configured Boeing 747-200B airplanes. The radio call sign when the president is aboard either aircraft or any Air Force aircraft is "Air Force One."

23. A mail-order company charges \$5 per order plus \$2 per item in the order, up to a maximum of 4 items. Write a function rule to describe the situation. Find a reasonable domain and range for the function.

24. **Transportation** Air Force One can travel 630 miles per hour. Let h be the number of hours traveled. The function rule $d = 630h$ gives the distance d in miles that Air Force One travels in h hours.

a. Identify the independent and dependent variables. Write $d = 630h$ in function notation.

b. What are reasonable values for the domain and range in the situation described?

c. How far can Air Force One travel in 12 hours?

25. Complete the table for $g(z) = 2z - 5$.

z	1	2	3	4
$g(z)$				

26. Complete the table for $h(x) = x^2 + x$.

x	0	1	2	3
$h(x)$				

27. **Estimation** For $f(x) = 3x + 5$, estimate the output when $x = -6.89$, $x = 1.01$, and $x = 4.67$.

28. **Transportation** A car can travel 30 miles on a gallon of gas and has a 20-gallon gas tank. Let g be the number of gallons of gas the car has in its tank. The function rule $d = 30g$ gives the distance d in miles that the car travels on g gallons.

a. What are reasonable values for the domain and range in the situation described?

b. How far can the car travel on 12 gallons of gas?

29. **Critical Thinking** Give an example of a real-life situation for which the reasonable domain consists of 1, 2, 3, and 4 and the reasonable range consists of 2, 4, 6, and 8.

30. **ERROR ANALYSIS** Rashid saves \$150 each month. He wants to know how much he will have saved in 2 years. He writes the rule $s = m + 150$ to help him figure out how much he will save, where s is the amount saved and m is the number of months he saves. Explain why his rule is incorrect.

31. **Write About It** Give a real-life situation that can be described by a function. Explain which is the independent variable and which is the dependent variable.

MULTI-STEP TEST PREP

32. This problem will prepare you for the Multi-Step Test Prep on page 260.

The table shows the volume v of water pumped into a pool after t hours.

a. Determine a relationship between the time and the volume of water and write an equation.

b. Identify the independent and dependent variables.

c. If the pool holds 10,000 gallons, how long will it take to fill?

Amount of Water in Pool

Time (h)	Volume (gal)
0	0
1	1250
2	2500
3	3750
4	5000

TEST PREP

33. Marsha buys x pens at \$0.70 per pen and one pencil for \$0.10. Which function gives the total amount Marsha spends?

Ⓐ $c(x) = 0.70x + 0.10x$ Ⓒ $c(x) = (0.70 + 0.10)x$

Ⓑ $c(x) = 0.70x + 1$ Ⓓ $c(x) = 0.70x + 0.10$

34. Belle is buying pizzas for her daughter's birthday party, using the prices in the table. Which equation best describes the relationship between the total cost c and the number of pizzas p?

Pizzas	Total Cost ($)
5	26.25
10	52.50
15	78.75

Ⓕ $c = 26.25p$ Ⓗ $c = p + 26.25$

Ⓖ $c = 5.25p$ Ⓙ $c = 6p - 3.75$

35. **Gridded Response** What is the value of $f(x) = 5 - \frac{1}{2}x$ when $x = 3$?

CHALLENGE AND EXTEND

36. The formula to convert a temperature that is in degrees Celsius x to degrees Fahrenheit $f(x)$ is $f(x) = \frac{9}{5}x + 32$. What are reasonable values for the domain and range when you convert to Fahrenheit the temperature of water as it rises from 0° to 100° Celsius?

37. **Math History** In his studies of the motion of free-falling objects, Galileo Galilei found that regardless of its mass, an object will fall a distance d that is related to the square of its travel time t in seconds. The modern formula that describes free-fall motion is $d = \frac{1}{2}gt^2$, where g is the acceleration due to gravity and t is the length of time in seconds the object falls. Find the distance an object falls in 3 seconds. (*Hint*: Research to find acceleration due to gravity in meters per second squared.)

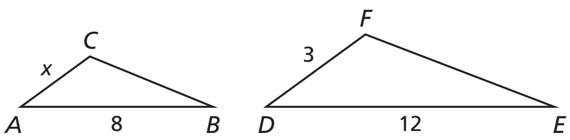
SPIRAL REVIEW

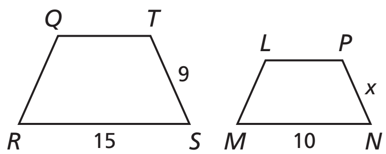
Solve each equation. Check your answer. *(Lesson 2-3)*

38. $5x + 2 - 7x = -10$ **39.** $3(2 - y) = 15$ **40.** $\frac{2}{3}p - \frac{1}{2} = \frac{1}{6}$

Find the value of x in each diagram. *(Lesson 2-7)*

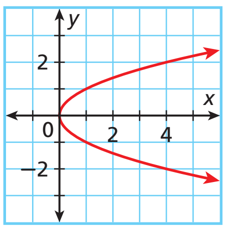
41. $\triangle ABC \sim \triangle DEF$ **42.** $QRST \sim LMNP$





Give the domain and range of each relation. Tell whether the relation is a function and explain. *(Lesson 4-2)*

43.



44.

x	y
−3	4
−1	2
0	0
1	2
3	−4